MESONIC FORM FACTORS IN THE LIGHT-FRONT QUARK MODEL

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Form factors for $P \to P$ and $P \to V$ transitions due to the valence-quark configuration are calculated directly in the physical time-like range of momentum transfer within the light-front quark model. It is pointed out that the Bauer-Stech-Wirbel type of light-front wave function fails to give a correct normalization for the Isgur-Wise function at zero recoil in $P \to V$ transition. Some of the $P \to V$ form factors are found to depend on the recoiling direction of the daughter mesons relative to their parents. Thus, the inclusion of the non-valence contribution arising from quark-pair creation is mandatory in order to ensure that the physical form factors are independent of the recoiling direction.

The q^2 dependence of the mesonic form factors is customarily assumed to be governed by near pole dominance in most existing approaches. In principle, QCD sum rules, lattice QCD simulations, and quark models allow one to compute form-factor q^2 behavior. However, the analyses of the QCD sum rule yield some contradicting results. For example, while $A_1^{B\rho}$ is found to decrease from $q^2 = 0$ to $q^2 = 15\,\mathrm{GeV}^2$ in [1], such a phenomenon is not seen in [2,3]. Also the sum-rule results become less reliable at large q^2 due to a large cancellation between different terms. The present lattice QCD technique does not allow to compute the form factors directly in weak B decays. Additional assumptions on extrapolation from charm to bottom scales and from $q^2_{\rm max}$ to other q^2 have to be made.

As for the quark model, the form factors evaluated in the non-relativistic quark model is trustworthy only when the recoil momentum of the daughter meson relative to the parent one is small. As the recoil momentum increases, we have to start considering relativistic effects seriously. In particular, at the maximum recoil point $q^2 = 0$ where the final meson could be highly relativistic, there is no reason to expect that the non-relativistic quark model is still applicable. A consistent treatment of the relativistic effects of the quark motion and spin in a bound state is a main issue of the relativistic quark model. To our knowledge, the light-front quark model [4] is the only relativistic quark model in which a consistent and fully relativistic treatment of quark spins and the center-of-mass motion can be carried out. In previous works [5,6] using $q^+ = 0$, one can only calculate form factors at q^2 =0; moreover, the form factors $f_-(q^2)$ in $P \to P$ decay and $a_-(q^2)$ in $P \to V$ decay cannot be studied. Hence extra assumptions are needed to extrapolate the form factors to cover the entire

range of momentum transfer. Based on the dispersion formulation, form factors at $q^2 > 0$ were obtained in [7] by performing an analytic continuation from the space-like q^2 region. Finally, the weak form factors for $P \to P$ transition were calculated in [8,9,10] for the first time for the entire range of q^2, so that additional extrapolation assumptions are no longer required. This is based on the observation [11] that in the frame where the momentum transfer is purely longitudinal, i.e., $q_\perp = 0$, $q^2 = q^+q^-$ covers the entire range of momentum transfer. The price one has to pay is that, besides the conventional valence-quark contribution, one must also consider the non-valence configuration (or the so-called Z-graph) arising from quark-pair creation from the vacuum.

For the first time, we have calculated the $P \to V$ form factors directly at time-like momentum transfers by evaluating the form factors in a frame where $q^+ \geq 0$ and $q_\perp = 0$. Our main results are [12]:

1). We have investigated the behavior of the form factors in the heavy-quark limit and found that the requirements of heavy-quark symmetry for $P \to P$ transition and for $P \to V$ transition are indeed fulfilled by the light-front quark model provided that the universal function $\zeta(v \cdot v')$ obtained from $P \to V$ decay is identical to the Isgur-Wise function $\xi(v \cdot v')$ in $P \to P$ decay.

2). Contrary to the Isgur-Wise function in $P \to P$ decay, the normalization of $\zeta(v \cdot v')$ at zero recoil depends on the light-front wave function used. We found that the Baure-Stech-Wirbel (BSW) [13] amplitude correctly gives $\xi(1) = 1$, but $\zeta(1) = 0.87$. Therefore, this type of wave functions cannot describe $P \to V$ decays in a manner consistent with heavy-quark symmetry. The incomplete overlap of wave functions at zero recoil in $P \to V$ transition implies spin symmetry breaking. In other words, when the spin-1 Melosh transformation acts on the BSW wave function, it will induce a spin-symmetry breaking term not suppressed by $1/m_Q$.

3). Using the Gaussian-type amplitude, the Isgur-Wise function $\zeta(v \cdot v')$ has a correct normalization at zero recoil and is identical to $\xi(v \cdot v')$ numerically up to six digits. It can be fitted very well with a dipole dependence with $M_{\rm pole} = 6.65$ GeV for $B \to D$ transition. However, the predicted slope parameter $\rho^2 = 1.24$ is probably too large, probably due to the lack of enough high-momentum components at large $k_\perp$ in the wave function.

4). The valence-quark and non-valence contributions to form factors are in general dependent on the recoiling direction of the daughter meson relative to the parent meson, but their sum should not. Although we do not have a reliable estimate of the pair-creation effect, we have argued that, for heavy-to-heavy transition, form factors calculated from the valence-quark configuration evaluated in the "+" frame should be reliable in a broad kinematic region, and they become most trustworthy in the vicinity of maximum recoil.

5). The form factors $F_1,\ A_0,\ A_2,\ V$ (except for $V^{B\rho}$ and V^{BK^*}) all exhibit a dipole behavior, which F_0 and A_1 show a monopole behavior in the close vicinity of maximum recoil for heavy-to-light transition, and in a broader kinematic region for heavy-to-heavy decays. Therefore, $F_1,\ A_0, A_2,\ V$ increase with q^2 faster than F_0 and A_1.

6). In the following we present some numerical results for form factors at maximal recoil $q^2 = 0$ evaluated in the "+" frame. We found [12]

$$F_1^{BD}(0) = F_0^{BD}(0) = 0.70\,, \tag{1}$$

and

$$f_+^{B\pi}(0) = 0.29\,,\quad f_+^{BK}(0) = 0.34\,,\quad f_+^{D\pi}(0) = 0.64\,,\quad f_+^{DK}(0) = 0.75\,, \tag{2}$$

for $P \to P$ transitions, and

$$V^{BD^*}(0) = 0.78\,,\ A_0^{BD^*}(0) = 0.73\,,\ A_1^{BD^*}(0) = 0.68\,,\ A_2^{BD^*}(0) = 0.61\,, \tag{3}$$

as well as

$$\begin{array}{llll} A_0^{BK^*}(0) = 0.32\,, & A_1^{BK^*}(0) = 0.26\,, & A_2^{BK^*}(0) = 0.23\,, & V^{BK^*}(0) = 0.35\,, \\ A_0^{DK^*}(0) = 0.71\,, & A_1^{DK^*}(0) = 0.62\,, & A_2^{DK^*}(0) = 0.46\,, & V^{DK^*}(0) = 0.87\,, \\ A_0^{B\rho}(0) = 0.28\,, & A_1^{B\rho}(0) = 0.20\,, & A_2^{B\rho}(0) = 0.18\,, & V^{B\rho}(0) = 0.30\,, \\ A_0^{D\rho}(0) = 0.63\,, & A_1^{D\rho}(0) = 0.51\,, & A_2^{D\rho}(0) = 0.34\,, & V^{D\rho}(0) = 0.78\,, \end{array} \tag{4}$$

for $P \to V$ transitions. Experimentally, only $D \to K^*$ form factors have been measured with the results [14]

$$V^{DK^*}(0) = 1.1 \pm 0.2\,,\quad A_1^{DK^*}(0) = 0.56 \pm 0.04\,,\quad A_2^{DK^*}(0) = 0.40 \pm 0.08\,, \tag{5}$$

obtained by assuming a pole behavior for the q^2 dependence. Our predictions for the $D \to K^*$ form factors are consistent with experiment. Two form-factor ratios defined by

$$\begin{aligned} R_1(q^2) &= \left[1 - \frac{q^2}{(M_B + M_{D^*})^2}\right] \frac{V^{BD^*}(q^2)}{A_1^{BD^*}(q^2)}, \\ R_2(q^2) &= \left[1 - \frac{q^2}{(M_B + M_{D^*})^2}\right] \frac{A_2^{BD^*}(q^2)}{A_1^{BD^*}(q^2)}, \end{aligned} \tag{6}$$

have been extracted by CLEO [15] from an analysis of angular distribution in $\bar{B} \to D^* \ell \bar{\nu}$ decays with the results:

$$R_1(q^2_{\rm max}) = 1.18 \pm 0.30 \pm 0.12\,,\qquad R_2(q^2_{\rm max}) = 0.71 \pm 0.22 \pm 0.07\,. \tag{7}$$

Our light-front calculations yield $V^{BD^*}(q^2_{\rm max}) = 1.14\,,\ A_1^{BD^*}(q^2_{\rm max}) = 0.83\,,$ and $A_2^{BD^*}(q^2_{\rm max}) = 0.96$, hence $R_1(q^2_{\rm max}) = 1.11$ and $R_2(q^2_{\rm max}) = 0.92\,,$ in agreement with experiment.

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